Analysis of Image Compression in Curvelet Domain

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ANALYSIS OF IMAGE COMPRESSION IN CURVELET DOMAIN

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ROURKELA, INDIA

May 2015

Declaration of Authorship

I, Dipshikha NARAYAN, declare that this thesis titled, " Analysis of Image Compression in Curvelet Domain" and the work presented in it are my own. I confirm that:

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This is to certify that the thesis entitled, **" Analysis of Image Compression in Curvelet Domain"** submitted by **Dipshikha Narayan** in partial fulfillment of the requirements for the award of Master of Technology Degree in **Electrical Engineering** with specialization in **Electronic Systems and Communication** during 2014-2015 at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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"If there's a book you really want to read, but it hasn't been written yet, then you must write it."

$\sim$ Toni Morrison

Abstract

Analysis of Image Compression in Curvelet Domain

Curvelet transformation is a multiscale representation of signals. It localizes signal in scale and position as wavelets, but it is the localization in orientation, which advances its performance in representing edges sparsely. Edges are the most crucial information in images, and efficient processing of the information containing data is the primary motivation for image compression processes. In this research work, Curvelet transform (CT) has been used in image compression. Even though, CT is a well-established mathematical tool, the literature suggests that no significant contribution has been in employing curvelets in image compression. In this work, the conventional compression standards like JPEG and JPEG2000 are studied extensively, and the procedure is extended and made compatible with curvelets. Experimental results show that the proposed method produces somewhat similar SSIM and PSNR values that of conventional standards, but improvement has noticed in the decrement of edge mismatch error but with poor performance in compression ratio. To mitigate this problem compressive sensing is used, which samples the signal in less number of samples and better sample reduction has been observed.

Acknowledgements

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Dipshikha Narayan

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Abbreviations

FT	**F**ourier **T**ransform
STFT	**S**hort **T**ime **F**ourier **T**ransform
WT	**W**avelet **T**ransform
DWT	**D**iscrete **W**avelet **T**ransform
CT	**C**urvelet **T**ransform
CS	**C**ompressive **S**ampling
SST	**S**hannon's **S**ampling **T**heorem
QM	**Q**uantization **M**atrix
ZT	**Z**ero **T**ree

Dedicated to My Village

1
Introduction

1.1 Introduction

"A PICTURE IS WORTH A THOUSAND WORDS", we always use this sentence. But the actual scenario is that in this modern days, an image cost million times more data than words. Images are rich in data. Our eyes even not able to distinguish these all data and more often these all are not even important for us. So it is a necessity to keep only that data which is important and discard other. It is the point through which image compression stem out and has become compulsory everywhere like the internet, digital TV, facsimile, voice messaging, etc.

In the realm of digital image compression, there are number of image compression standard available. In which JPEG (Joint Photographic Experts Group) and JPEG2000 are the most widely used image compression standards. Every compression standard works to preserve relevant information. The main difference in all compression standards are the transform used. As JPEG [1] uses DCT, at the same place, JPEG2000 takes DWT. It has seen that for continuous tone images, JPEG works in a better manner but when it come to sharp-edged images then it fails. So, as an advance technique JPEG2000 was evolved. With a multiscale architecture, DWT provides a better performance with less distortion.

But JPEG2000 also have shortcomings, as with every subband decomposition of an image through DWT, with approximation part it preserves only three detail. But now, an advanced version of WT is available known as Curvelet transform. It also has multiscale architecture but at the same time it give detail(edge) information in many orientations.

1.2 Motivation

Edge, the sudden variation of intensity in a spatial image are the most delicate and valuable information. The enormous number of researchers are going to do compression efficiently, everywhere the efficiency of an image compression standard has defined in terms of representing image information in fewer numbers of data. To representing the edge information sparsely, curvelet transform surpasses the Fourier, Discrete cosine or Wavelet transforms. So, in this work image compression in curvelet domain is analysed with the expectation that even after compression up to a large extent, a compressed image will be rich in detail information.

TABLE 1.1: Literature review

Auther	**Year**	**Contribution**
David A. Huffman[3]	1952	Huffman coding
N. Ahmed, T. Natarajan and K. R. RAO[6]	1972	DCT
Gregory K. Wallace [1]	1991	Explained about JPEG
J. M. Shapiro [2]	1993	Embedde Zerotree Wavelet coding
Emmanuel Candes and David Donoho[4]	2002	Contineous Curvelet Transform
Emmanuel Candes, Laurent Demanet,David Donoho and Lexing Ying[5]	2006	Fast Discrete Curvelet Transform

1.3 Objective

- Understand the mechanism of image compression
- Study of JPEG and JPEG2000
- Analysis of Curvelet transform
- Analyse Curvelet transform in image compression
- Compare proposed compression technique with other existing compression standard

1.4 Contributions of the thesis

- Curvelet transform has implemented in image compression
- Compressive sampling is an efficient technique for sampling, which is rarely used by traditional compression standards

1.5 Thesis overview

This thesis is splited into six chapters:

- Chapter 1 and 2 talks about the need and fundamental ideas about image compression. Chapter 2, covers the some important terms related to image compression and the image compression block.
- Chapter 3, presents brief idea about different transforms like DCT, DWT and Curvelet transform. It also discusses Huffman coding that has been by many of the standard compression standards. In the last part, there is a little discussion about Compressive sampling and one very important metric Edge mismatch.
- Chapter 4 deals with JPEG and JPEG2000.
- Chapter 5 introduces the proposed algorithm designed for image compression. In this chapter, image are compressed with JPEG, JPEG and through the proposed method that follows the general image compression steps. Here advantages and shortcomings of proposed method are highlighted. That failure is solved by introducing the compressive sampling in the last section of this chapter.
- The last Chapter 6 concluded with the contributions of this thesis and discussing the future work and future scope regarding this work.

2

Background of Image Compression

2.1 Image Compression

IMAGE COMPRESSION IS THE PROCESS of reducing the amount of data, required to represent a specific quantity of application dependent information. In today's electronic era, the data sharing, storage and manipulation become very easy but the thing that blocks our freedom is the data storage size and its transfer rate. But one thing is very clear that whatever data or message we work with, all not contains information. There are lots of data that unnecessarily present in our system or disk, known as **Redundant data**. Image compression works on to remove these redundant data while preserve the valuable information containing data.

2.1.1 Redundant data

In any image mainly three types of redundant data do present, they are explained as follows :

- **Spatial redundancy:** Spatial redundancy presents in almost every image. There is an enormous correlation between the adjacent pixels or in other word hardly any pixel changes its intensity from its neighbourhood pixel. So many times the image is always full of unnecessary repetition of same intensities. Data, which are only repeating the some data without causing any extra information are called spatial redundant data.

- **Psychovisual redundancy**:Psychovisual redundancy present in an image due to the inaccuracy of our eye. In every image application, the final observer is a human and human visual system function like a low pass filter. That is human eye can readily distinguish the change in intensity if it changes occur for long time but fails to recognize the very slight variation between homogeneous surfaces or intensities.

- **Coding redundancy:** In the gray level images, eight bits are required to represent single pixel intensity. But in images, hardly every gray level intensities do present, and even if they all will be present, hardly they will be equiprobable. So, to still representing the pixels with 8 bits will be the wastage of bits.

2.1.2 Terms in Image Compression

There are mainly two terms in image compression, which gives information about extent of compression and amount of redundancy removed. They are given as follows[11]

- **Compression Ratio (CR) :** It tells about number of times an original image is larger than its compressed version of image in terms of bits.

$$C = \frac{b}{b'} \tag{2.1}$$

  Where, b is the number of bits in orignal image and b' is the number of bits in compressed image.

- **Relative Data Redundancy (R) :** It tells about percentage of redundant data which has discarded after compression of an original image.

$$R = 1 - \frac{1}{C} \tag{2.2}$$

2.2 Image Compression Model

Every image compression system comprises of two specific functional blocks[11] : Encoder and Decoder.

Encoder is used to do compression in which compressed intensities transformed into bit form.

- Encoding starts with mapper (transform), and it provides an ease to the compression processes. Mapper is used to decorrelate image data which is a reversible operation. As well as provide a better energy compaction property in the transform domain. After mapping, the image intensities that was earlier in the spatial domain will be projected in the transformed domain. Mapper can be DCT or DWT, depends on the Compression standard used.

- The quantizer is used to remove psychovisual redundancy. It is the primary cause of loss in lossy image compression standards, sometimes that losses will not be reversible. But it can be reversible in case of lossless coding.

- Symbol encoder is a reversible operation, used to remove the coding redundancy. Aim in symbol encoding is to map the quantized intensities in bit form.

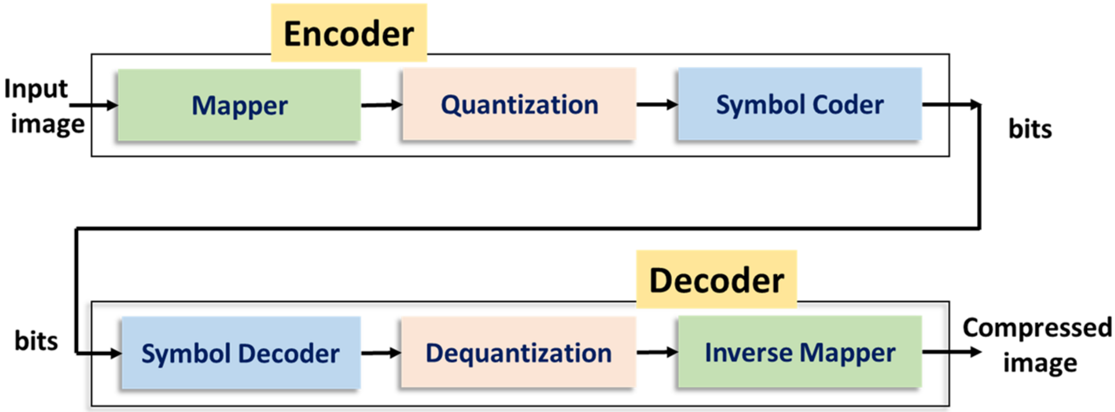

FIGURE 2.1: Block diagram of Image compression standards

Decoder Decoder performs the inverse process of an encoder in reverse order i.e. whatever process has been done in last step of encoding, their inverse operation will be done in the first step of an encoder and vice versa. After decoding as an output we get compressed spatial image.

3

Tools for Compression

3.1 Transform

TRANSFORM IS THE most important mathematical tool in the area of signal processing. We process the signal to highlight or suppress some characteristic of the signal. For this, signal get manipulated many times. As to get clarity of speech signal we need to discard noise from it. And if an old person is watching television then we do increase the volume of speaker or if it is a rainy day, then our dish antenna feels problem in catching the signals. Sometimes interference, distortion or due to our requirement, we need to change the signal. But we cannot change the signal directly in the form we receive it. As it is very difficult to process the signal in its original form. For example, if we need to remove noise from a speech signal, then we need to remove the high-frequency term. For this, we need to change signal in the frequency domain. Transform does the same process, it project signal from one domain to other domain. There are many transforms available, e.g. Fourier transform, Cosine transform, Wavelet transform, Curvelet transform, etc. For every transform, the purpose is same to project signal in other domain, and it is the energy compaction property that make them arrange signal differently.

3.1.1 Cosine Transform

DCT [6] was co-invented by N. Ahmed, T. Natarajan and K. R. Rao in 1973. Its basis function is orthogonal sinusoid like Fourier basis and it is simpler than Fourier transform. The forward DCT transform can be written as the dot(inner) product of input signal and orthogonal sinusoids as in the equation given below[11],

$$T(u,v) = \sum_{x=0}^{N}\sum_{y=0}^{N} g(x,y)r(x,y,u,v) \tag{3.1}$$

for u, v = $0, 1, 2, ...n-1$ and basis image :

$$r(x,y,u,v) = \alpha(u)\alpha(u)cos\left[\frac{(2x+1)u\pi}{2N}\right]cos\left[\frac{(2y+1)v\pi}{2N}\right] \tag{3.2}$$

where

$$\alpha(u) = \alpha(v) = \begin{cases} \sqrt{\frac{1}{n}} & \text{if } u = 0, \\ \sqrt{\frac{1}{2n}} & \text{if } u = 1, 2, 3, ..., n-1 \end{cases} \tag{3.3}$$

where, $g(x, y)$ is an image of size $N \times N$, $r(x, y, u, v)$ is the transformation kernel. $T(u, v)$ is the spatial image intensities $g(x, y)$ in wavelet domain.

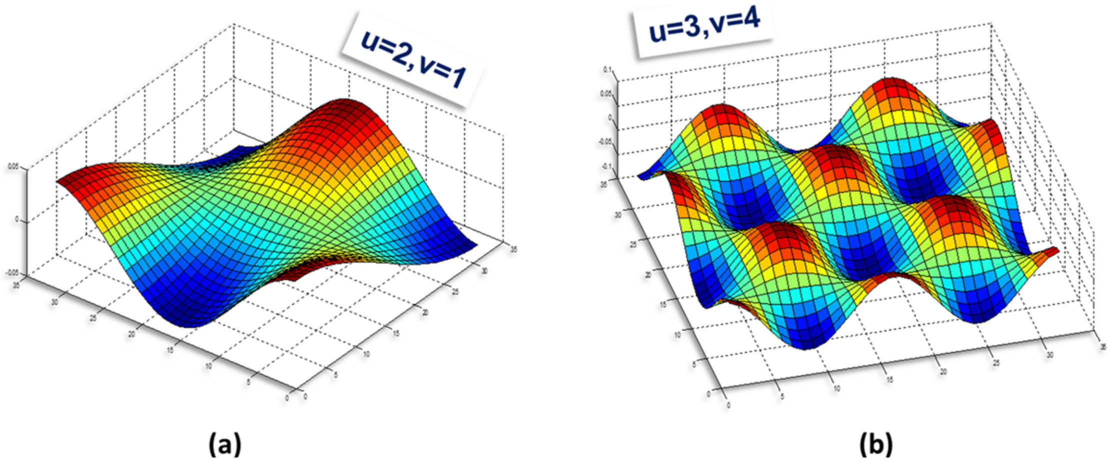

FIGURE 3.1: **DCT basis function :** In both the basis images (a) and (b), dimension of the transformed coefficient is 32-by-32. As the values of u and v is increasing, frequency of the basis is also increasing, basis having high frequency, fetches high frequency components of the image

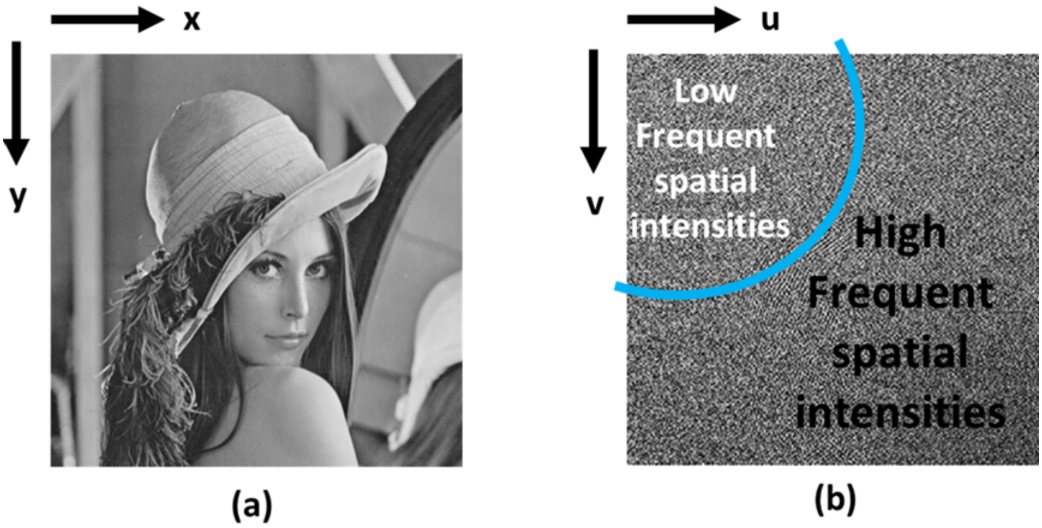

FIGURE 3.2: **DCT transformed coefficient :** (b) is the transformed image of (a); (b) retains the frequency components shown in the figure, But it doesn't localizes the spatial frequencies

If we apply DCT on a block of an image, then information of that image block will be transformed into the frequency domain. In DCT, all the coefficients will be the representative of the image block in the spatial domain. But, we cannot precisely pinpoint which frequency in DCT domain present at which spatial location of an image.

So there was a need to develop shorter frame. Short time fourier transform (STFT) easily used to localize the frequency components. But, STFT has also some shortcoming, as it uses sinusoidal frequency as basis, which is extended from -∞ to +∞. And in STFT there is need to forcibly truncate this basis. Due to which in the reconstructed signal some windowing effect will occur which disturb the performance of the signal up to a large extent.

So, there was a need to go where we can take space-frequency localization simultaneously. Then WT was evolved as a solution to the above problem.

3.1.2 Wavelet Transform

The wavelet transform is an invertible, lossless transform. It uses a small wave of limited duration, and varying frequency called $wavelets$ to project the signal.

Theory of Wavelet [11] :

It says that if a $f(x) \in L^2(R)$, then this function can be expressed as :

$$f(x) = \sum \alpha_k \varphi_k(x) \tag{3.4}$$

where,α is a constant, $\varphi(x)$ is expansion function and k is integer index of summation. If $f(x) \in V$, φ_k closely span the function $f(x)$, and can be realized using scaled and shifted versions of some basic function.

As WT has multiresolution architecture, it uses different scale (decimation and upsampling) and space (position) of Scaling function $\varphi_{j,k}(x)$ and Wavelet function $\psi_{j,k}(x)$ to reconstruct the signal. For a efficient representation of signal, scaling function takes only one scale (j_0) where as wavelet function takes different scale depends on the user.

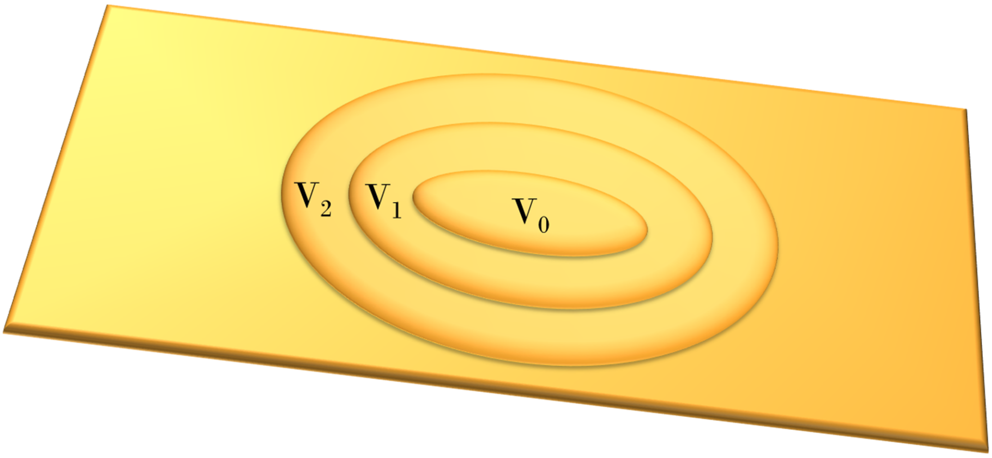

FIGURE 3.3: Subspace spanned by scaling function

Representation of signal by scaling function is like low pass filtering of signal and same by wavelet function is like high pass filtering. Scaling function at the low scales span the subspaces that are the subset of subspaces spanned by scaling functions at higher scales. A wavelet function $\Psi(x)$ are defined that can also be scaled and translated from the basic wavelet function. Wavelet function covers the difference between any two neighbouring scaling function V_j and V_{j+1} subspace, which is W_j.

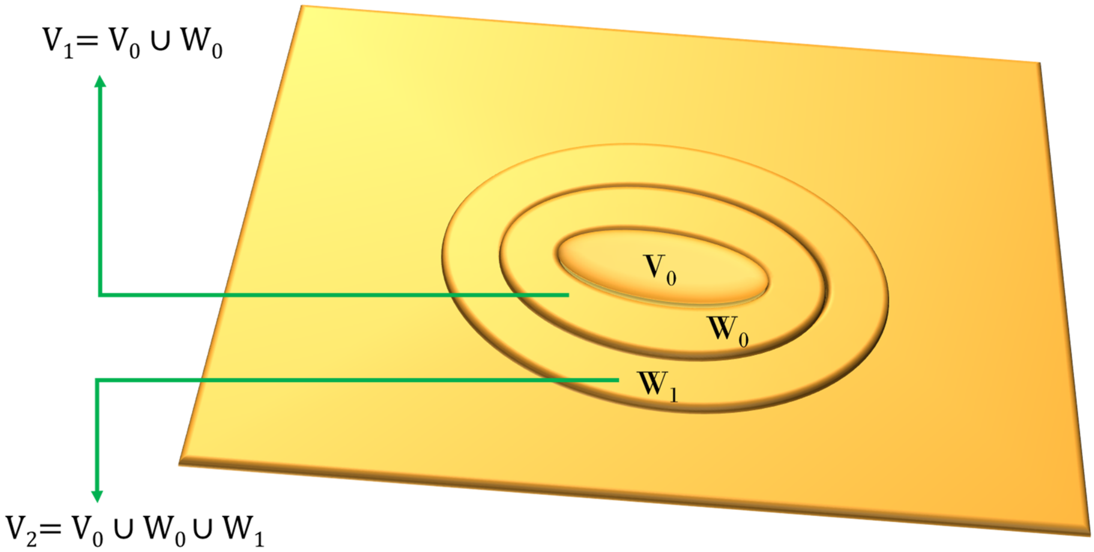

FIGURE 3.4: Subspace spanned by wavelet function

3.1.2.1 Discrete Wavelet Transform on Image

Application of DWT on 2D signals is like the application of low and high pass filter on both the dimension of an image. After that, the image get decomposed into one approximation and three detail coefficients. Three detail coefficients will be horizontal, vertical and diagonal details.

$$W_{\varphi}(j_0, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\varphi_{j_0,m,n}(x,y) \tag{3.5}$$

where, $W_{\varphi}(j_0, m, n)$ is the approximation subbands at an arbitrary scale j_0.

$$W^i_{\psi}(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\psi^i_{j,m,n}(x,y), ...i = H, V, D \tag{3.6}$$

where, $W^i_{\psi}(j, m, n)$ is the horizontal (H), vertical (V) and diagonal (D) details.

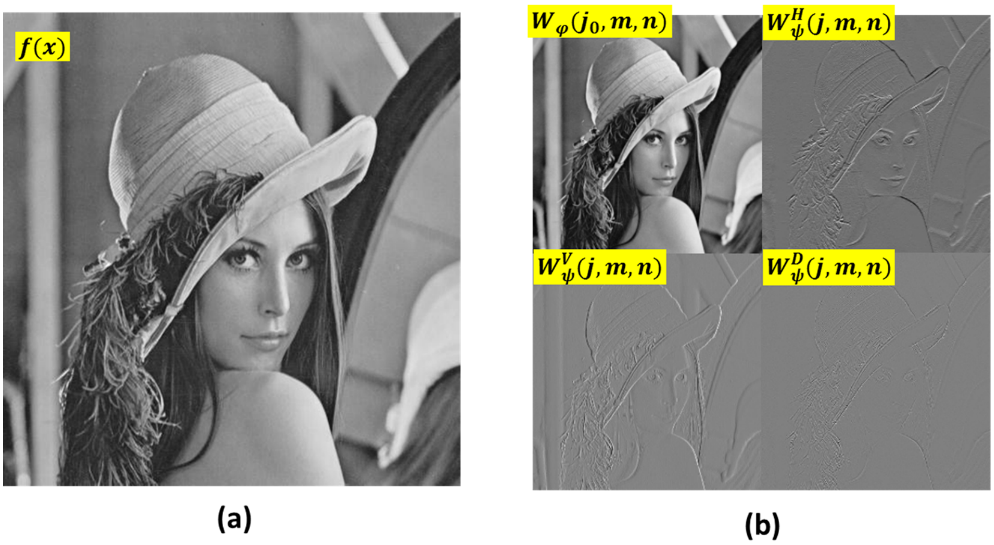

FIGURE 3.5: **DWT :** It decomposes (a) into (b); which is a combination of approximation and three detail subbands, but as unalike as DCT, localizes the spatial frequencies

DWT can be efficiently implemented on the concept of a filter bank. The first decomposition gives the finest subbands and in next decomposition, the approximation coefficients get divided into four subbands, and this process of

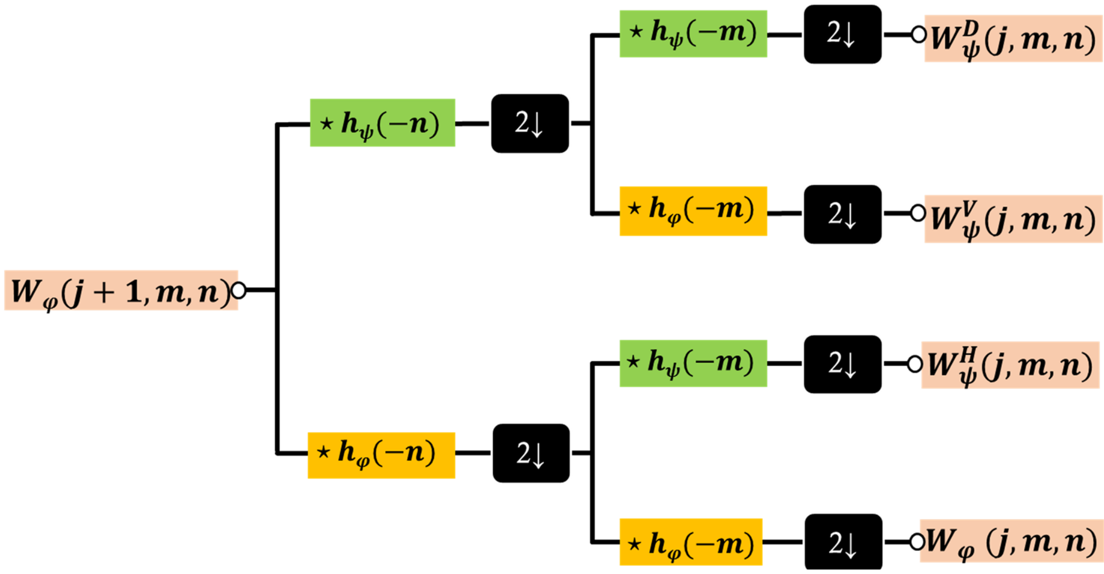

FIGURE 3.6: 2-D fast DWT analysis filter bank

decomposition of approximation goes on till the target decomposition will be reached.

3.1.2.2 Inter Relation between Wavelet subbands

As per each decomposition, an image is getting splitting up into four subbands. And the bandwidth of the spatial bandwidth of the signal gets $\frac{1}{4}^{th}$ of the total image frequency. So as a result of that we will be getting redundant samples with no loss in information by throwing out the alternate samples in both the dimension.

As the motive of this whole discussion is to understand the mechanism of DWT from the image compression point of view. In compression, by transform redundancy is exploited. In WT, redundancy is exploited in the same pixel location but in successive subbands, which gives an excellent space-frequency localization of image pixels.

In WT, if we are working at one pixel of an image, means we are working at one spatial location but at different resolution i.e. at different scales of wavelet filtered coefficients.

As resolution is increasing ,number of pixels are increasing, one thing is to notice here in figure 4.6 one pixel at LL_3 subband corresponds to one pixel of

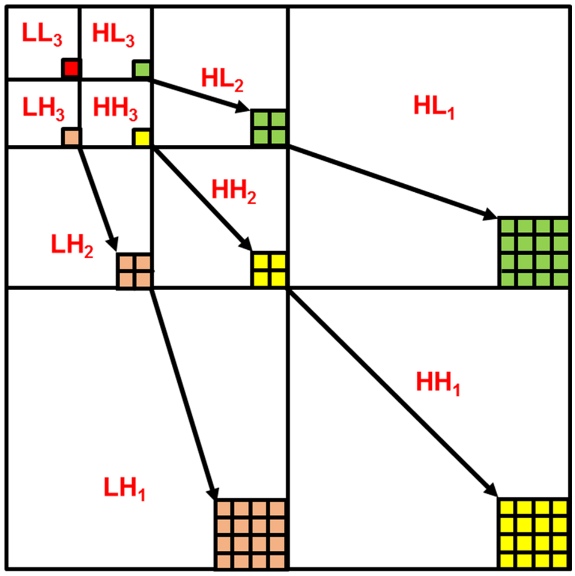

FIGURE 3.7: **DWT subbands relationship :** Most lowest frequent coefficients will be situated on the top-left corner and high frequencies will be near down-right corner. At a particular location of spatial image, pixels in each subband will be spatially corresponds to four times of that no.s of pixels to the next lowest scale subband of same detail

HL_3, LH_3 and HH_3 at the same pixel position. These all three will have four descendants in HL_2, LH_2 and HH_2 subbands respectively and simultaneously 16 descendants at HL_1, LH_1, and HH_1 respectively.

In this way, WT coefficients are related in different subbands. As we are going to higher and higher resolution, we are getting low significant coefficients. This nature of multi-resolution WT provides an ease to image compression, which we will discuss later in chapter4.

3.1.3 Curvelet Transform

Multiresolution architecture of Curvelet transform provides a new dimension to the multidimensional theory. In wavelet domain, the image get decomposed into approximation and detail subbands of different scale, lower than the scale or original image. Other than scale and space there is one more signal localization property that presents in curvelet, make it different from the other.

In the figure, Curvelets are shown in the spatial domain, at a different scale, space and orientation. The direction of the shear is called as **Diagonal direction** and along the perpendicular direction of shear is called as **Off-diaginal direction**. We can see from the figure given below, frequencies are varying along the off-diagonal direction as well as smooth along the diagonal direction.

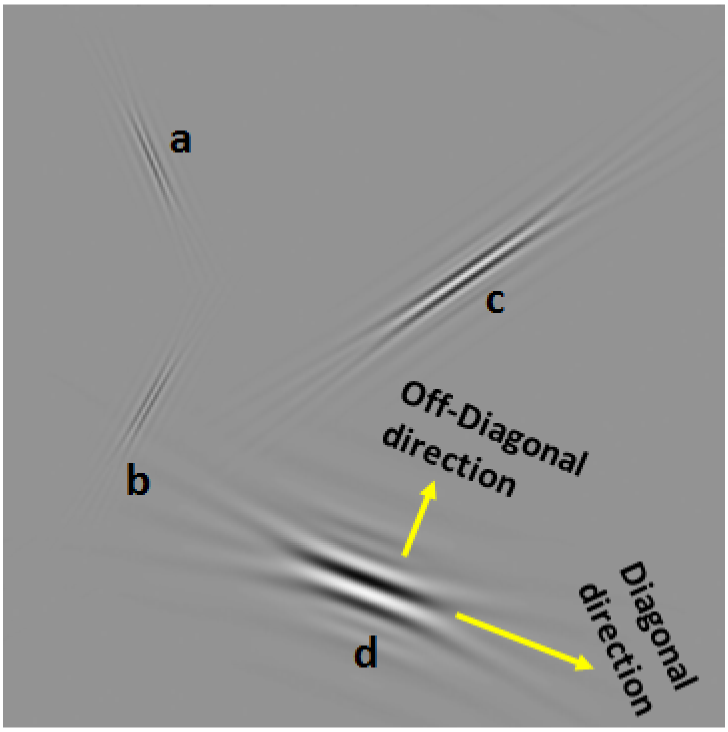

FIGURE 3.8: Curvelet in spatial domain

As the resolution increases, curvelet becomes fine as well as small in the spatial domain and efficiently traces the edges. Curvelet becomes of needle shape at the finer scale. Each non-zero curvelets span a wedge shape area in the frequency domain, and this wedge dimension follows the parabolic scaling law. Fine curvelets cover more area in frequency axis.

Curvelet analyses image in a much efficient way than wavelets. Curvelet support is anisotropic in nature and follow parabolic scaling law $length^2 \approx width$. CD contains a multiresolution architecture, here is some point regarding curvelets :

- Curvelet gives an ideal representation for curve singularity
- It also gives exact reconstruction and follows Perceval's relation
- It is like wavelet with one more localization parameter of orientation that's why called as geometrical wavelets or directional wavelet.

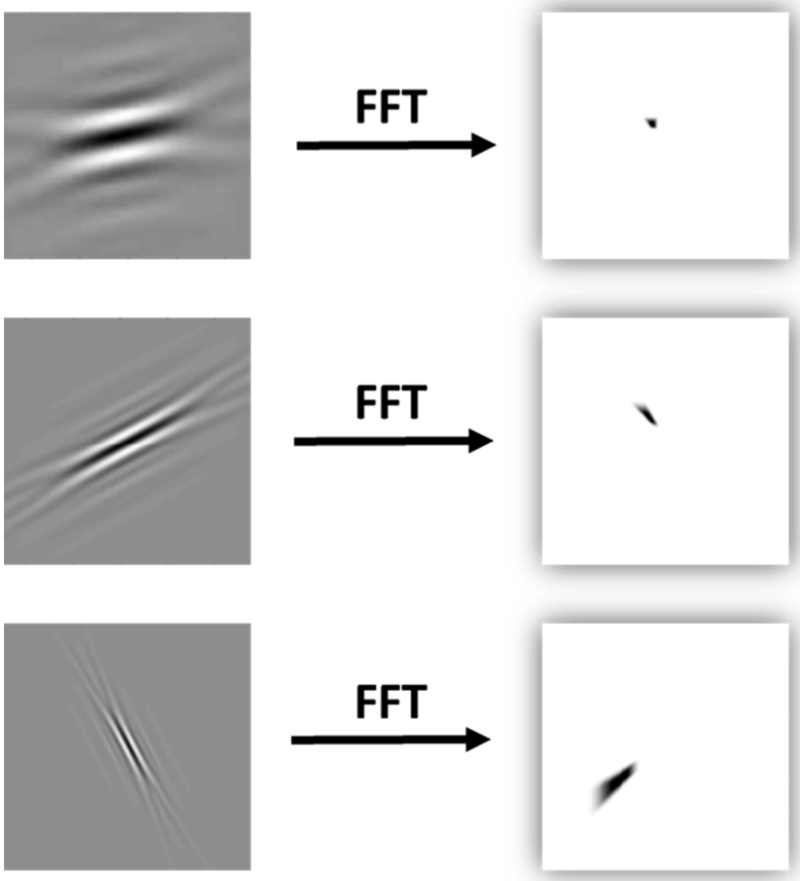

FIGURE 3.9: Curvelets and its fourier transform

- Curvelet project the frequency components of an image into dyadic Cartesian coronae (concentric squares and shears) and subdivide them into wedges according to the orientation as well as parabolic law.
- At the fine scale curvelet is of a needle shape and traces signal in a better way but at coarser scale it loses its directionality. As wavelet gives the space-frequency localization of the spatial signal in the same way curvelets give space-phase (scale and orientation) space-phase localization.

3.1.3.1 Continuous Time Curvelet Transform

Continuous time curvelet transform[13] for 2D signal $f \in L^2(R^2)$ can be defined as the inner product between f and continuous curvelet waveform $\varphi_{j,l,k}$;

$$\varphi_{j,l,k} =< f, \varphi_{j,l,k} >= \int_{R^2} f(x)\overline{\varphi_{j,l,k}(x)}dx. \tag{3.7}$$

As, WT can be defined by scaling and wavelet function, in the same way full CT can be defined at fine scale directional element $\varphi_{j,l,k_{j \geqslant j_0,l,k}}$ and at coarser scale an isotropic father wavelets $(\Phi_{j_0,k})_k$.

3.1.3.2 Digital Curvelet Transform

Digital curvelet transform for a 2D signal $f(m,n)$ for $0 \leq m < M$, $0 \leqslant n < N$ is defined as :

$$C^D(j,l,k) = \sum_{0 \leq t_1,t_2 < n} f[t_1,t_2]\overline{\varphi^D_{j,l,k}[t_1,t_2]} \tag{3.8}$$

Here, $\varphi^D_{j,l,k}(t_1,t_2)$ is the digital curvelet waveform. Transformed coefficients is a function of scale (j), space (k) and orientation (l). A fast and digital platform is provided in 2005-2006, named as **second generation of curvelets**. In this generation, two methods was provided based on :

- **Unequally Spaced Fast Fourier Transform (USFFT)**, and
- **Wraping**

Steps for implementation of Curvelet via USFFT is given as follows:

- Take 2D FFT of input signal f
- Resample f for each scale and angle
- Multiply the resampled signal with the parabolic window
- Apply 2D IFFT to each of the subsample taken after third step to get the discrete curvelet coefficients $C^D(j,l,k)$

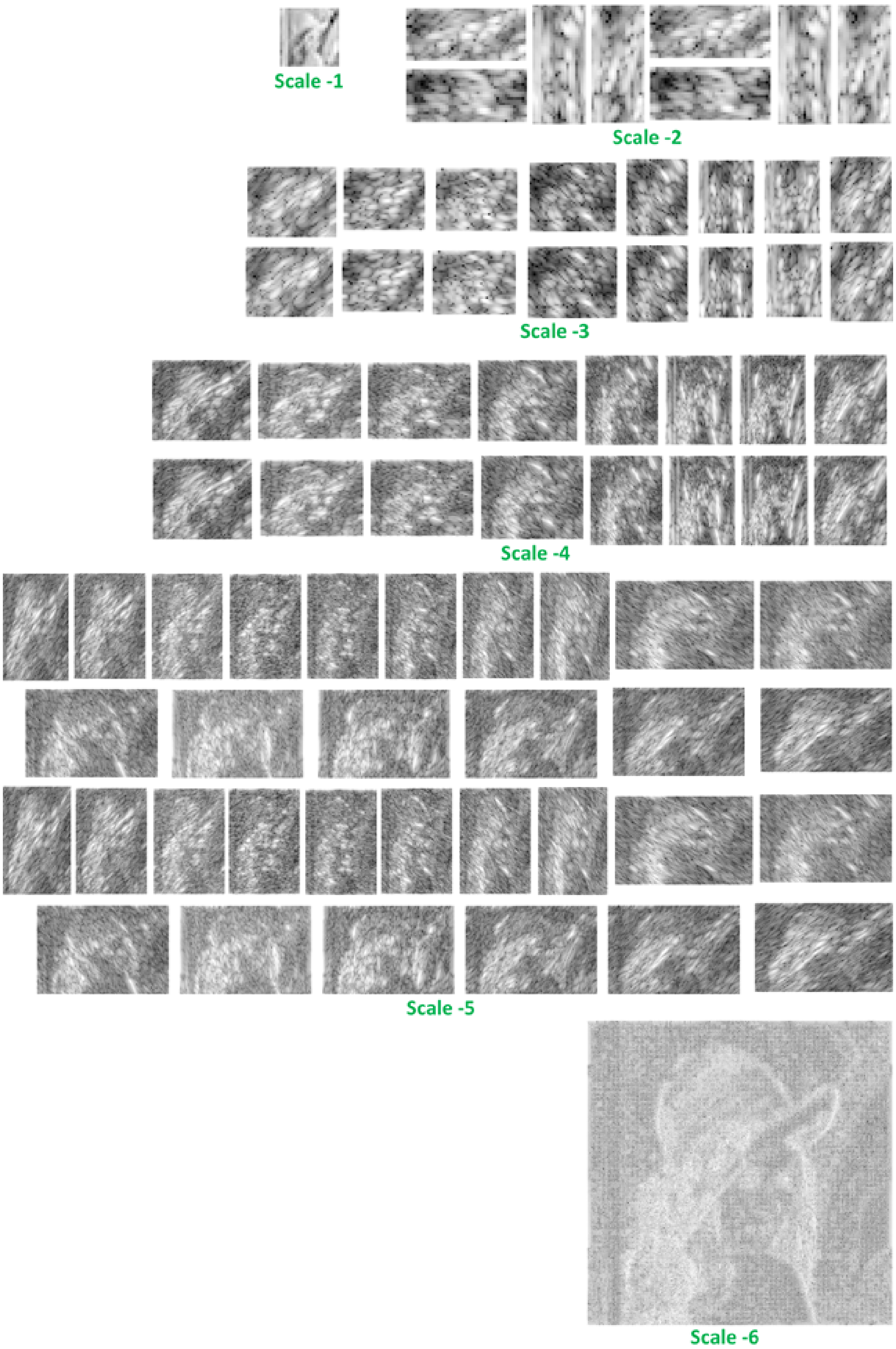

FIGURE 3.10: **Discrete Curvelet transformed coefficients** of image in figure 3.5 (a), into six scales and in different orientations, detailed information about matlab function and subbands are given in table 3.1 and 3.2 respectively

TABLE 3.1: Parameters used in transformation of image in curvelet domain

Curvelab Version	CurvelaLab 2.1.3	
Website	www.curvelab.org	
Matlab file	fdct_wrapping.m	
Input image (X)	Lena.jpg	Graylevel image with the dimension of 512×512
is_real	0	Complex Curvelet Coefficients
finest	2	Wavelet at the finest level
nbscales	6	No. of levels of decimposition
nbangles_coarse	8	No. of orientations at the second coarsest level

TABLE 3.2: Number of scale and orientation subbands

Scale	**No. of orientations**
1	1
2	8
3	16
4	16
5	32
6	1

Steps for implementation of Curvelet via Wrapping is given as follows:

- Take 2D FFT of input signal f
- Subdivide the frequency spectrum and multiply each subband to the image spectrum for their respective scale and angle and make then in a wedge shape
- Wrap the wedges into rectangular region
- Apply 2D IFFT to each of the rectangular wedge after and get the discrete curvelet coefficients $C^D(j, l, k)$

As CT is a well developed mathematical tool, it is used in many areas of image processing like image denoising, image fusion, image contrast enhancement, etc. But the area of image compression is still untouched.

3.2 Huffman Coding

Huffman Coding (HC) is one of the most famous and optimal coding redundancy removal technique. This method was given by Huffman[3] in 1952. It represent symbols in terms of bits and at the same time allot the less possible number of bits to each symbol of symbol source.

There are some constraints on HC, which provides an ease to the coding method [3] :

1. Every symbol has provided distinct set of bits
2. Here only position we need is the starting point of the sequence, once it is known, no extra bits should be wasted to indicate where message code begins and ends,
3. If there are total N symbols are present to encoded and if their probability is like this :

$$P(1) \geqq) P(2) \geqq P(3) \geqq \ldots\ldots\ldots\ldots\ldots \leqq P(N) \tag{3.9}$$

   Then, length of the bitcode should be like this :

$$L(1) \leqq L(2) \leqq L(3) \leqq \ldots\ldots\ldots\ldots\ldots = L(N) \tag{3.10}$$

4. If in N symbols, D symbols are distinct. Then there will be not less than two and at most D of the symbols with code-length $L(N)$ have code bits, which will be similar except for the last digit.
5. Each possible sequence of one less than the highest word length $(L(N)-1)$ digits must be used as a prefix to the message code.

Coding Method :

HC works on the Source Reduction Technique . Subsequent steps for coding is given as follows:

1. Rearrange the symbols in the way of diminishing probability

2. Assign 0 and 1 to two least probable symbol, combine their probability and treat them as one symbol, now number of symbols will be reduced by one

3. Repeat step 2 till only two symbol will be left and assign them 0 and 1

4. For each one symbol code will be assigned as prefix to the previous source symbol

3.3 Compressive Sampling

Compressive Sampling (CS) provides a new type of signal sampling theory. The concept of Shannon's sampling theory (SST) came into the limelight 76 years later to SST in 2004 by Candès, Terence Tao, and David Donoho. Purpose of the CS is same as that of SST, but the mathematics behind the CS is entirely different from SST, even it violates the SST. As one place where SST depends on the highest frequency present in the signal. At the same place, CS depends on the sparsity of the signal.

Mathematical formulation of Compressive Sampling :

As in the compressed sample, dimension is required to less than the input signal length. So, this sampling problem is algebraically summarized as below :

$$\Phi_{M\times N} * X_{N\times 1} = Y_{M\times 1} \tag{3.11}$$

As, number of equations are less than the number as unknown, so these are under determined linear equation with the infinite solution. But CS theory works to get the optimal solution from the set of infinite solution.

Signal Recovery :

CS is based on recovering signal X through Convex Optimization. Here signal recovery equation is given based on L2-minimization :

$$\hat{x} = \arg \min_{y=ax} \|x\|_2 \tag{3.12}$$

$$\|x\|_0 < l \tag{3.13}$$

where, l is a constraint on the number of non-zero samples.

3.4 Edge Mismatch

Edge mismatch(EMM) algorithm works on the principal of matching the edge pixel of original and its processed image. If value of EMM is zero, means there is no mismatching of edge and if it is one i.e. edges are completely mismatched.

EME is an important performance metrics in the field of image compression, as it checks edges, which is the most crucial information of an image. Edge mismatch[9] metric can be expressed as follows :

$$\mathrm{EME} = 1 - \frac{\mathrm{CE}}{\mathrm{CE} + \omega\left[\sum_{k\in\{\mathrm{EO}\}} \delta(k) + \alpha \sum_{k\in\{\mathrm{ET}\}} \delta(k)\right]} \tag{3.14}$$

with,

$$\delta(k) = \begin{cases} |d_k| & if\ |d_k| < \mathrm{maxdist} \\ D_{max} & \mathrm{Otherwise} \end{cases} \tag{3.15}$$

Where,

CE	: No. of common edge pixels found between the reference image and the processed image
EO	: Set of excess reference image edge pixels missing in the processed image
ET	: Set of excess thresholded edge pixels not taking place in the reference image
ω	: Penalty associated with an excess original edge pixel
α	: Ratio of penalty associated with an excess processed image edge pixel to an excess original edge pixel
d_k	: Euclidean distance of the k'th excess edge pixel within a search area determined by maxdist parameter

It has been suggested to select the parameter as maxdist=$0.025N$, where N is the image dimension, $D_{max} = 0.1N$, $\omega = 10/N$, and $\alpha = 2$.

4

Study of Existing Compression Standards

4.1 Joint Photographic Experts Group

JPEG IMAGE COMPRESSION STANDARD IS[1] most widely used lossy baseline coding system for continuous-tone still images. In this standard DCT is used as mapper. As for the image of size $N \times N$ the computational complexity of DCT is $O(N^4)$, to lessen it, the algorithm is designed to divide an image into small non-overlapping blocks. For quantization, JPEG has designed a unique quantization matrix (QM), which will be of the same size as that of the image block. Each entry in the QM is psychovisually motivated to the human visual system. These entries are called stepsize (threshold) of the respective position of DCT coefficient. This QM is used to divide each image block, and quotient values are taken after the rounding operation. As in transformed image, coefficients near the top right corner are most visually significant. So to keep important image data, entries in QM near the top-right corner are designed as small in comparison to values at other positions are large and especially near bottom right corner it contains much larger values. For symbol encoding JPEG uses Huffman coding , which is based on source reduction technique.

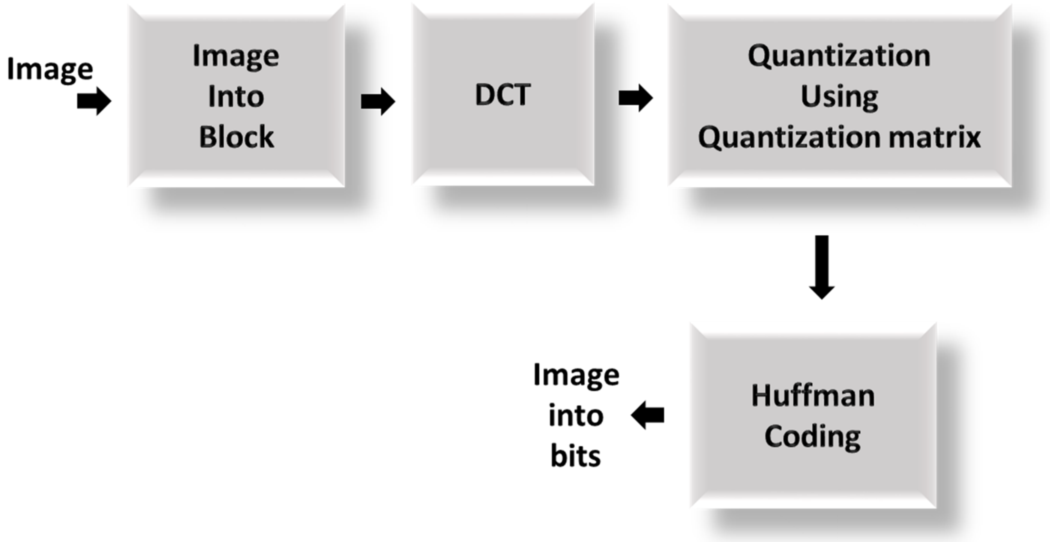

FIGURE 4.1: Block diagram of JPEG2000

Artifacts present in JPEG compression :

$$\begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

FIGURE 4.2: Quantization Matrix [1]

- **Blocking Artifacts :** As JPEG standard suggests to divided image into small non-overlapping blocks before transforming into DCT domain. But when quantization matrix is scaled by an integer value in case to get low quality compressed image, then in decoder end when all blocks will be merged to form the image. Then the local property of the image from block to block will get changed, which will be very much visible, they are called as blocking artifacts.

- **losing details:** Due to energy compaction property of DCT high-frequency components from an image can be easily removed by quantization. But when an image will be rich in high frequencies, then we can lose detail in the high-frequency part of the original so that the image will look blurred, and lack of features will occur.

These are some serious drawbacks of JPEG, then an extended version of it, called JPEG2000 was evolved.

4.2 JPEG2000

JPEG2000 Standard was developed from the same organization ISO/IESC/ITU-T as that of JPEG, with the promise of attaining the high compression ratio while maintaining the valuable information in comparison to JPEG. The main difference lies between compression standards is the transform used. JPEG uses DCT where JPEG2000 takes DWT.

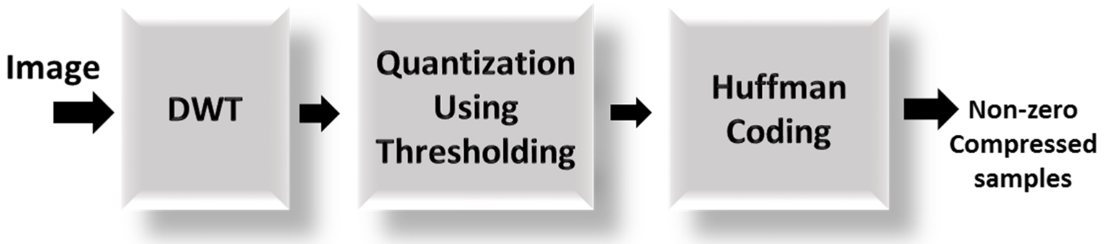

FIGURE 4.3: Block diagram of JPEG2000

4.2.1 Basic Concept of Compression in JPEG2000

As DWT has multiscale architecture, at each decomposition image get divided in approximation and three detail subbands. From figure4.6, we can conclude that in DWT coefficients a tree structure evolves. Means if we wanted to pick any one pixel from the spatial image. Then for that same spatial location, we will need to pick one pixel from the topmost LL subband. Then LL_n will be the root of the tree, that root corresponds to four subbands, where each subbands again will have four subband and it will continue till the highest scale.

To compress the wavelet coefficients, Jerome M. Shapiro [2], proposed a very efficient coding algorithm called as Embedded Zerotrees Wavelet algorithm (EZW). In EZW coding, encoding of intensities can be stopped at any point till the target bitrate will be achieved. After EZT, while extending its concept, lots of coding paradigm has evolved. Most of the other wavelet coding method are extended version of EZW. So, here we will take a little glimpse of EZW.

In wavelet decomposition, the image get divided into approximation subbands, and three other detail called horizontal, vertical and diagonal. With total

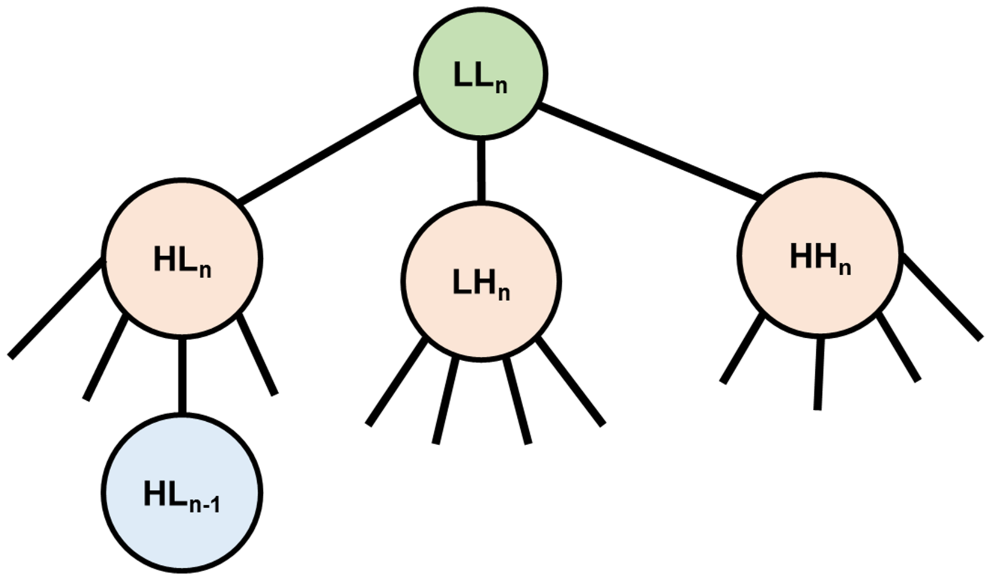

FIGURE 4.4: **Tree structure[2] of Wavelet coefficients :** Here LL_n is parent of coefficients at HL_n, LH_n and HH_n. LL_n is ancestor to its descendant, where as descendants are HL_n, LH_n HH_n and other that detail coefficients as finer scale.

N decomposition number of subbands will be $3N + 1$. The first decomposition gives the finest subbands then in next decomposition, the approximation coefficients get divided into four subbands, and this process goes on till the target decomposition will be reached.

For compression using EZW algorithm, a new data structure was formed, known as Zero Tree (ZT)[2]. For a wavelet coefficient c, will be treated as insignificant for a given threshold (T) if $|c| < T$. The ZT concept is based on a hypothesis, according to that wavelet coefficient at a coarser scale will be significant with respect to a given threshold T. Then all wavelet intensities for the detail at the same spatial location for finer scale will be insignificant for the same threshold T. Observations suggest that this hypothesis is often true.

For forming the zerotree some nomenclature was made:

- The coefficient at coarser scale is called **parent**. Which corresponds to all the coefficients at the same spatial location with the same detail type in the next finer scale, coefficients at that next fine subband is known as **child** of that parent.

- For a parent, all coefficients at a fine scale of same detail type and at the same spatial position are called as **descendants**.
- For a set of descendants, group of intensities at the coarser scale are called as **ancestors**.
- Scanning order should be like so that none of the children are accessed before their parent.
- For an N-level decomposition, scanning starts from LL_N. And after that it goes to the subbands HL_N, LH_N, and HH_N. Then it will move to the next scale i.e. N-1, etc.

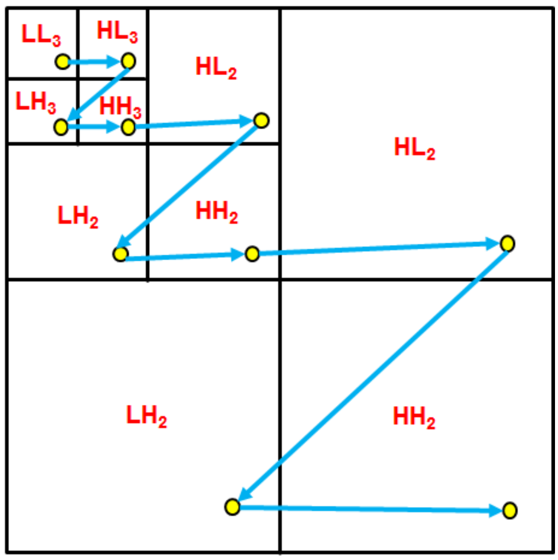

FIGURE 4.5: **DWT subband bit selection order :** Highest scaled subband contains the much low frequent detail. In a constraint bit budget, if scanning order of intensities is like specified in the figure above then image will be rich in valuable information

Significant coefficients are decided by comparing the magnitude of the coefficients with a T. Threshold are readjusted at every pass. In the beginning, a threshold value is chosen by the higher magnitude of the wavelet transformed coefficients. All coefficients whose absolute vale will be less than T will be treated as insignificant and the coefficients whose absolute value will be more than T, that will be retained. With the successive passes, the threshold value is halved.

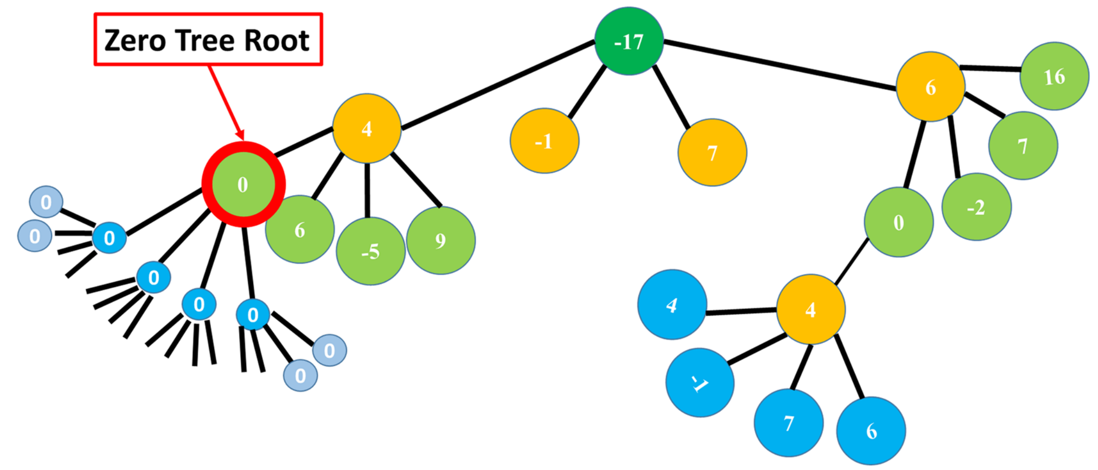

FIGURE 4.6: Zero tree of DWT

$$\begin{aligned} |c| &\leq T\text{......insignificant} \\ |c| &\geqslant T\text{.........significant} \end{aligned} \tag{4.1}$$

Zerotree is a part of the tree structure. Shapiro [2] suggested that if a parent is non-significant and all of its descendants are also zero. Then that subtree whose parent is at the root is called as zero tree, and that root is called zero tree root. Once a zero tree is found in particular pass then in that detail, coding should be stopped. In this way, in a limited bit budget we can retain the most important bits.

5

Proposed Compression method

5.1 Proposed Image Compression Block

IMAGE COMPRESSION METHOD proposed in this research work has followed the general compression block. One major difference is of the transform used, and here Curvelet transform is employed.

FIGURE 5.1: Curvelet based Image compression block

5.2 Quantization Algorithm Used

For Quantization thresholding based quantization is used. Here, two types of thresholding is used **Scale independent** and **Scale wise** thresholding. For scale dependent, half of the magnitude of largest transformed coefficient is taken as threshold. Whereas in scale wise, for each scale threshold value is decided as the half of the highest transformed coefficient at that scale. For source encoding huffman[3] coding is used.

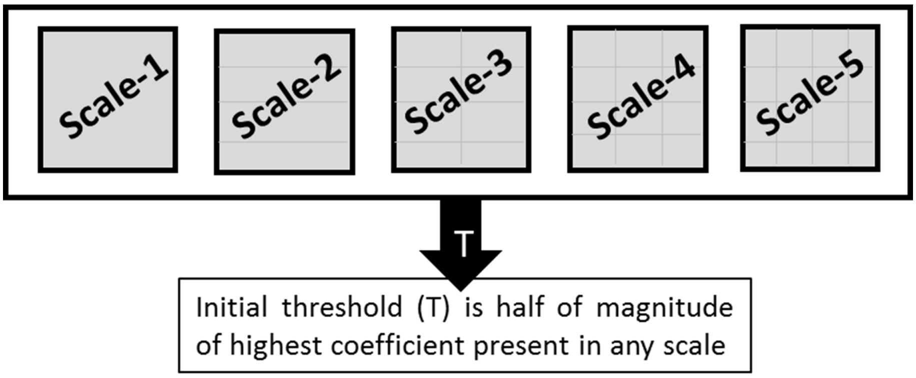

FIGURE 5.2: Way of Quantization : Scale independent

After quantization in curvelet domain it is observed that, the same hypothesis which was applicable for DWT, is true here also. That is, after quantization

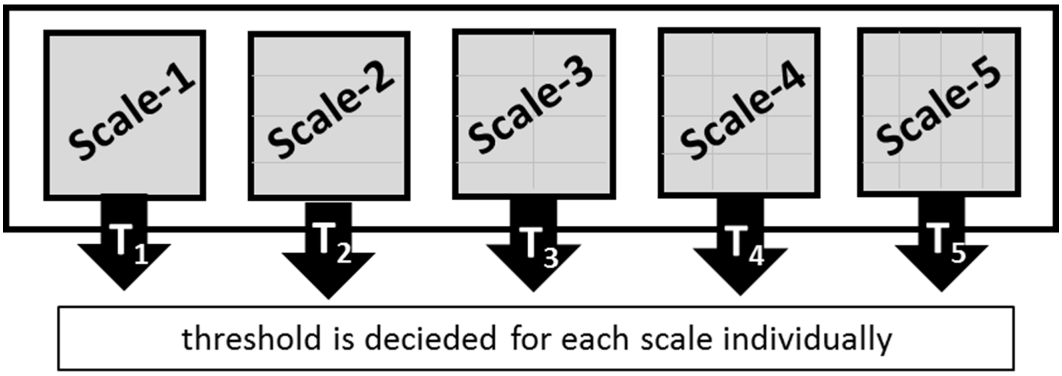

FIGURE 5.3: Way of Quantization : Scale wise

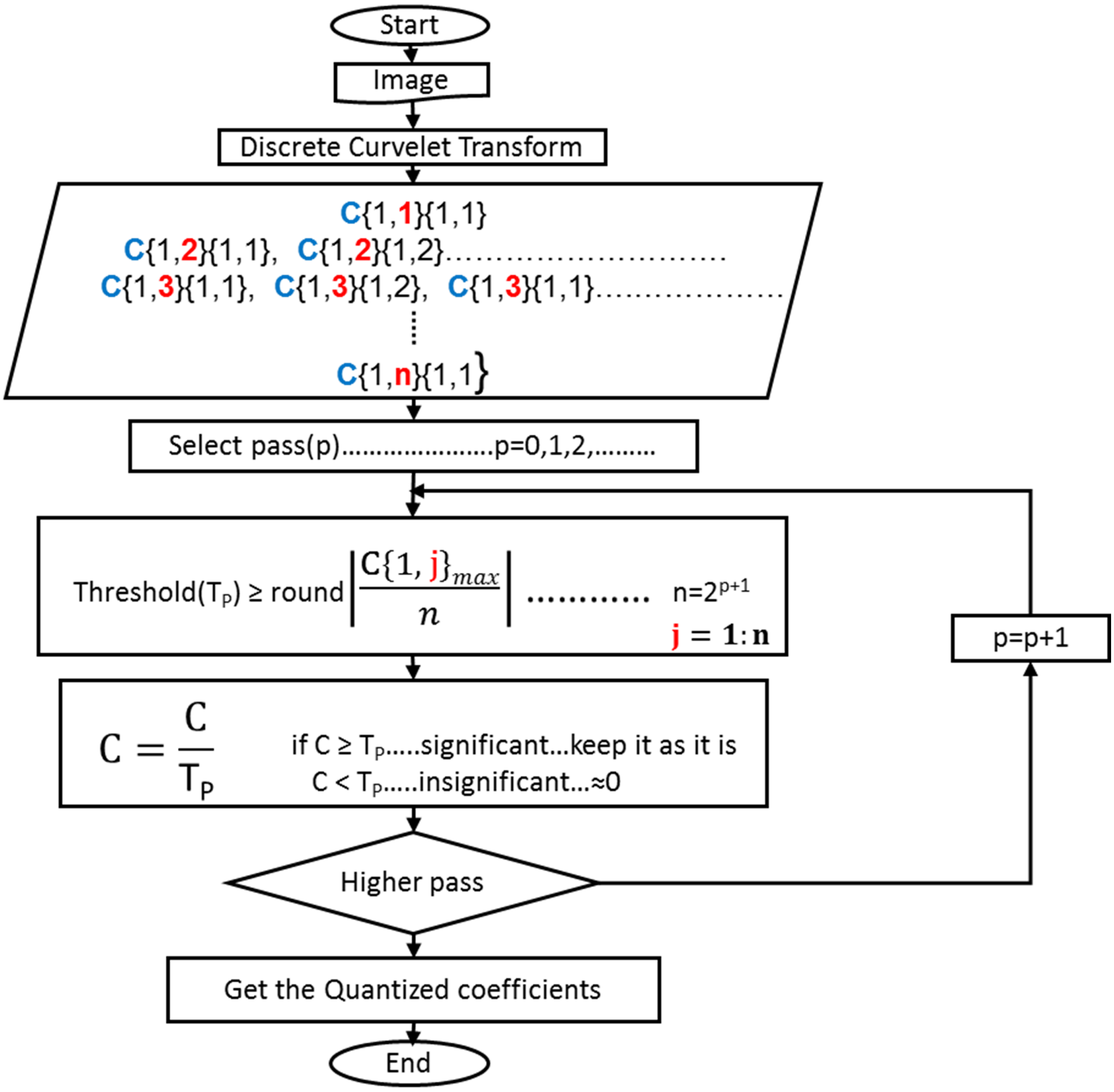

FIGURE 5.4: Proposed Quantization Algorithm

most of the coefficients at the low scaled subbands are becoming zero in comparision to high scaled subbands.

5.3 Compression Analysis

By the proposed algorithm image is compressed by keeping the both way of thresholding scale independent and scale wise. At different threshold value the CR, R , SSIM, PSNR, EMM and RZC are measured after quantization and given in table below. For curvelet transformation, codes are taken from [14].

where, full form of all the performance indices are given in the table below :

TABLE 5.1: Full form of the performance indices used

C_{max}	Largest transformed coefficient
CR	Compression Ratio
R	Relative data redundancy
SSIM	Structural Similarity Index Measurement
PSNR	Peak Signal to Noise Ratio
EMM	Edge Mismatch
RZC	Residual no. of Zero Coefficients after quantization

TABLE 5.2: Result of Proposed Method scale independent

PROPOSED COMPRESSION METHOD (SCALE INDEPENDENT)						
Threshold Value	$\frac{C_{max}}{2}$	$\frac{C_{max}}{4}$	$\frac{C_{max}}{8}$	$\frac{C_{max}}{16}$	$\frac{C_{max}}{32}$	$\frac{C_{max}}{64}$
CR	1.6331	.78292	.51156	.34579	.22651	.15488
R	38.7673	-27.7277	-95.4787	-189.1915	-341.4899	-545.6766
SSIM	.7113	.84909	.93666	.96959	.98368	.99229
PSNR	20.2167	25.9878	31.4171	35.2121	39.3817	43.8741
EMM	.4851	.34917	.26468	.22943	.20855	.19415
RZC	669890	651715	633897	613855	573978	504119

From TABLE 5.1 and TABLE 5.2 , we can conclude that scale independent thresholding than scale independent up to threshold $\frac{C_{max}}{2}$, but from threshold value $\frac{C_{max}}{4}$ scale wise thresholding is providing better result in terms of SSIM, PSNR and EMM.

TABLE 5.3: Result of Proposed Method scale wise

PROPOSED COMPRESSION METHOD (SCALE WISE)						
Threshold Value	$\frac{C_{max}}{2}$	$\frac{C_{max}}{4}$	$\frac{C_{max}}{8}$	$\frac{C_{max}}{16}$	$\frac{C_{max}}{32}$	$\frac{C_{max}}{64}$
CR	1.431	.53954	.28792	.1853	.12967	.098271
R	30.1177	-85.343	-247.3225	-439.6765	-671.1624	-917.5944
SSIM	.71451	0.8601	.95145	.98135	.99193	.99757
PSNR	20.2938	26.6048	34.1357	39.8338	44.6103	49.7913
EMM	.51568	.40321	.2619	.21048	.19952	.18522
RZC	668665	642345	601539	544063	460019	344420

5.4 Compression Comparision with Other Standards

Here compression through proposed compression method is compared with the standard compression methods JPEG and JPEG2000. For JPEG at different quality factor [7], performance is calculated. For JPEG2000 at different Quantization vector[8], performance is calculated. Codes of JPEG and JPEG2000 are taken respectively from [11] and [12].

As these three compression technique JPEG, JPEG2000 and proposed, all are very much different from each other in terms of transform used as well as the quantization algorithm used. One shortcoming of proposed algorithm is compression ratio is not effective, even the compressed image form is much larger in size, in comparison to original image. So, we need to fix some parameter for comparison. As, it is expected that curvelet efficiently represent curve of an image. So, for comparision, EMM of compressed image is compared with the JPEG and JPEG2000 compressed image, images whose SSIM and PSNR are nearly equal.

TABLE 5.4: Result of JPEG compression

JPEG						
Quality Factor	**10**	**20**	**30**	**40**	**50**	**60**
C	4.0729	2.555	1.8799	1.7588	1.1624	1.012
R	75.4472	60.8603	46.7756	43.1429	13.9718	1.1886
SSIM	.8977	.9517	.97017	.97661	.9803	.99174
PSNR	30.6395	33.5518	35.2294	36.1097	.36.4822	40.4784
EMM	.30744	.25505	.34275	.33424	.25681	.21932
RZC	252144	246448	240360	240353	226494	218030

TABLE 5.5: Result of JPEG2000 compression

JPEG2000							
Quantization Vector $[q_0\ q_1]$	[8, 7]	[8, 7.5]	[6.5, 8]	[6.5, 8.5]	[6.5, 9]	[6.5, 9.5]	[7, 10]
C	31.1903	21.2268	14.5933	10.0709	7.0272	4.9731	3.588
R	96.7939	95.289	93.1475	90.0704	85.7695	79.8918	72.1293
SSIM	.66556	.7169	.77115	.82082	0.86167	.89862	.92472
PSNR	23.0029	24.1571	25.3719	26.7632	28.3945	29.9191	31.3837
EMM	.58628	.46555	.41903	.37745	.30947	.27273	.24813
RZC	278620	278170	277524	276575	275228	273310	270682

For comparison for thresholding upto $\frac{C_{max}}{4}$, compressed image are taken into consideration from Scale independent thresholding and for other thresholding images are taken from scale wise thresholding.

Here we can see that Curvelet compressed image retaining more detail information. As per the above mentioned condition.

TABLE 5.6: Comparision between JPEG and Proposed method

JPEG		Proposed compression method	
SSIM	**EMM**	**SSIM**	**EME**
.97017	35.2294	.95145	.2619
.97661	.33424	.95145	.2619
PSNR	**EMM**	**PSNR**	**EMM**
36.1097	.33424	34.1357	.2619
40.4784	.21932	39.8338	.21048

TABLE 5.7: Comparision between JPEG2000 and Proposed method

JPEG2000		Proposed compression method	
PSNR	**EMM**	**PSNR**	**EMM**
23.0029	.58628	20.2167	.4841
26.7632	.37745	25.9878	.34917

As we have seen that the proposed compression method is not effective as compressed image form is coming much larger in size than the JPEG and JPEG2000 compression. But in curvelet transformed image no. of coefficients come much more than the no. of spatial intensities

As well as, there is one more problem of random runs of zeros. After quantization, we can get coefficients with the abundance of zeros. But these zeros are randomly distributed at random in numbers. For this a special run length coding is used, where it is needed to remember the starting and ending position of zeros. To remove this memory constraint, compressive sampling is employed in this work, where positions of any types of intensities not required to remember.

5.5 Compressive sampling: As an alternative approach

FIGURE 5.5: CS based proposed Image compression block

As a solution to the problem mentioned above, CS is used to reduce the number quantized coefficient. For this, a greedy algorithm Orthogonal Matching Pursuit (OMP) has used. For OMP codes are taken from [15] We can see the modified curvelet based compression block.

TABLE 5.8: Number of non zeros

Threshold Value	$\frac{C_{max}}{2}$	$\frac{C_{max}}{4}$	$\frac{C_{max}}{8}$	$\frac{C_{max}}{16}$	$\frac{C_{max}}{32}$	$\frac{C_{max}}{64}$
Conventional method	19463	45783	86589	144065	460019	344420
Compressive Sampling	320	420	420	420	420	420

We can see here that image whose initial number of coefficients was 688128, by the application of proposed quantization algorithm and CS, get reduced very much, which is much better than JPEG and JPEG2000.

6

Concluding remarks and future work and scope

CT is used as mapper in proposed compression method. We have seen that compressed image for nearly the same SSIM and PSNR values are retaining most of the edge information. But limitation present here was the random runs and the Curvelet transform itself. The problem in the transform is that samples in transform image is much more than the samples in the original image. But this is not a case in DCT and DWT, as samples in compressed and non-compressed are nearly equal. So to mitigate these problem Compressive sampling is used in the place of symbol encoding, which gives a considerable reduction in number of samples.

CS is used at the place of symbol encoding. Analysis, work is done in this research work, so next work is the reconstruction of the original signal from the reduced sample is to do.

Future work of this work can be :

To get the more relevant information with keeping less distortion in compressed. Like in DWT each coefficient at different subbands are not only spatial related in numbers of intensities. If this type of property i.e. how many intensities in different scales and orientations will be determined will be developed the compression will be very efficient and compression will be adapted to the bit budget.

Bibliography

[1] G. K. Wallace, "Embedded image coding using zerotrees of wavelet coefficients", *IEEE Transactions on Signal Processing*, vol. 41, no. 12, pp. 3445–3462, 1993.

[2] J. M. Shapiro, "The JPEG still picture compression standard", *IEEE Transactions on Customer Electronics*, vol. 34, no. 4, pp. 30–44, 1992.

[3] D. A. Huffman, "A method for the construction of minimum redundancy codes", *Proceedings of the IRE*, vol. 40, no. 9, pp. 1098–1101, 1952.

[4] E. J. Candes and D. L. Donoho, "Continuous curvelet transform: I. Resolution of the wavefront set", *Journal of Applied and Computational Harmonic Analysis*, vol. 19, no. 2, pp. 162–197, 2005.

[5] E. J. Candes, L. Demanet, D. L. Donoho, and L. Ying, "Fast discrete curvelet transforms", *Journal of Multiscale Modeling and Simulation*, vol. 5, no. 3, pp. 861–899, 2006.

[6] N. Ahmed, T. Natarajan, D. L. Donoho, and K. R. Rao, "Discrete cosine transform", *IEEE Transactions on Computers*, vol. 100, no. 1, pp. 90–93, 1974.

[7] D. G. W. Onnasch, G. P. Prause, and A. Ploger "Quantization table design for JPEG compression of angiocardiographic images", *5th Intern Sym Coronary arteriography* , Rotterdam, June 28–30, 1993.

[8] M. Rabbani, R. Prause, and R. Joshi, "An overview of the JPEG 2000 still image compression standard", *Journal on Signal processing: Image communication*, vol. 17, no. 1, pp. 3-48, 2002.

[9] M. Sezgin and B. Sankur, "Comparison of thresholding methods for non-destructive testing applications", *IEEE ICIP'2001, Intl. Conf. Image Process*, pp. 764–767, 2001.

[10] D. Taubman and M. W. Marcellin, "JPEG2000 Image Compression Fundamentals, Standards and Practice", *Book*, vol. 1, 2002.

[11] R. C. Gonzalez and R. E. Rafael, "Digital image processing", *Book*, Third Edition, Pearson Education India, June 28–30, 2012.

[12] R. C. Gonzalez, R. E. Rafael and S. L. Eddins, "Digital image processing using MATLAB", *Book*, Pearson Education India, 2004.

[13] M. Kamel and A. Campilho, "Image Analysis and Recognition", *Book*, Springer Science and Business Media, 2009

[14] E. Candes and et. al. Curvelab 2.1.3 [Online]. Available: http://www.curvelet.org/software.html.

[15] J. Mairal and et. al. spams-matlab v2.2 [Online]. Available: http://spams-devel.gforge.inria.fr/downloads.html.

[16] M. Davenport, "Compressive sensing and Signal reconstruction", *Video Lecture* [Online]. Available: http://www.brainshark.com/brainshark/brainshark.net/portal/title.aspx?pid=zCdz10BfTRz0z0.